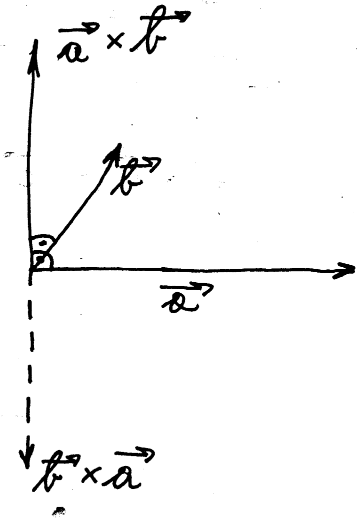
Vektorski proizvod

(dva vektora)

$\vec{a}\cdot\vec{b} \equiv$ realan broj

$\vec{a}\times\vec{b} \equiv$ vektor

$|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\cdot\sin\sphericalangle(\vec{a},\vec{b})$



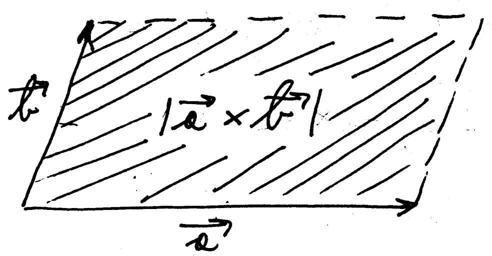
$$\vec{a}\times\vec{b}\perp\vec{a}$$

$$\vec{a}\times\vec{b}\perp\vec{b}$$

$$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$$

$\vec{a}\parallel\vec{b} \iff \vec{a}\times\vec{b} = 0$

ovo je uslov kolinearnosti dva vektora



$P_{\square} = |\vec{a}\times\vec{b}|$

$\vec{a}(a_1, a_2, a_3)$

$\vec{b}(b_1, b_2, b_3)$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

1. Dati su vektori $\vec{a}=(0, 2\lambda, \lambda)$, $\vec{b}=(2,2,1)$ i $\vec{c}=(-1,-2,-1)$. Odrediti vektor $\vec{d}$ koji zadovoljava uvjete

$$\vec{a}\times\vec{b} = \vec{c}\times\vec{d} \quad i \quad \vec{a}\times\vec{c} = \vec{b}\times\vec{d}.$$

Rj.
$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2\lambda & \lambda \\ 2 & 2 & 1 \end{vmatrix} = (2\lambda-2\lambda)\vec{i} - (0-2\lambda)\vec{j} + (0-4\lambda)\vec{k}$$
$$= (0, 2\lambda, -4\lambda)$$

Neka je vektor $\vec{d}=(x, y, z)$

$$\vec{c}\times\vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & -1 \\ x & y & z \end{vmatrix} = (-2z+y)\vec{i} - (-z-x)\vec{j} + (-y-2x)\vec{k}$$

kako je $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ to je

$$\begin{aligned}-2z+y&=0\\ z+x&=2\lambda\\ -2x-y&=-4\lambda\end{aligned}$$

$$\begin{aligned}y&=2z\\ z+x&=2\lambda\\ -2x-y&=-4\lambda\end{aligned}$$

$$\begin{aligned}y&=2z\\ x+z&=2\lambda\\ -2x-2z&=-4\lambda \quad /:(-2)\end{aligned}$$

$$\begin{aligned}y&=2z\\ x+z&=2\lambda \quad \ldots(*)\end{aligned}$$

$$\vec{a}\times\vec{c}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ 0&2\lambda&\lambda\\ -1&-2&-1\end{vmatrix}=(-2\lambda+2\lambda)\vec{i}-(0+\lambda)\vec{j}+(0+2\lambda)\vec{k}=(0,-\lambda,2\lambda)$$

$$\vec{b}\times\vec{d}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ 2&2&1\\ x&y&z\end{vmatrix}=(2z-y)\vec{i}-(2z-x)\vec{j}+(2y-2x)\vec{k}=$$
$$=(-y+2z,\ x-2z,\ -2x+2y)$$

kako je $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ to je

$$\begin{aligned}-y+2z&=0\\ x\quad -2z&=-\lambda\\ -2x+2y\quad&=2\lambda\end{aligned}$$

$$\begin{aligned}y&=2z\\ x-2z&=-\lambda\\ -2x+4z&=2\lambda \quad /:(-2)\end{aligned}$$

$$\begin{aligned}y&=2z\\ x-2z&=-\lambda \quad \ldots(**)\end{aligned}$$

Iz $(*)$ i $(**)$ dobijemo

$$y=2z$$
$$-\begin{cases}x-2z=-\lambda\\ x+z=2\lambda\end{cases}$$

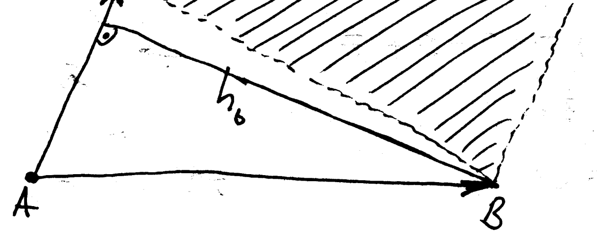
$$\begin{aligned}y&=2z\\ -3z&=-3\lambda\end{aligned}$$

$$y=2z$$
$$z=\lambda,\quad y=2\lambda,\quad x=\lambda$$

Vektor $\vec{d}$ je $\vec{d}(\lambda,2\lambda,\lambda)$.

2. Naći površinu i visinu koja odgovara stranici AC trougla $\triangle ABC$ ako je $A(-3,-2,0)$, $B(3,-3,1)$ i $C(5,0,2)$.

R.



$A(-3,-2,0)$, $B(3,-3,1)$ $\Big\}$ $\vec{AB}(6,-1,1)$

$C(5,0,2)$ $\quad \vec{AC}(8,2,2)$

$$P_{\square}=|\vec{AB}\times\vec{AC}|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -1 & 1 \\ 8 & 2 & 2 \end{vmatrix} = (-2-2)\,\vec{i} - (12-8)\,\vec{j} + (12+8)\,\vec{k} = (-4,-4,20)$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{16+16+400} = \sqrt{432} = \sqrt{16 \cdot 27} = \sqrt{4^2 \cdot 3^2 \cdot 3} = 12\sqrt{3}$$

$$P_{\Delta ABC} = \frac{|\vec{AB} \times \vec{AC}|}{2} = 6\sqrt{3}$$

$$P_{\Delta ABC} = \frac{|\vec{AC}| \cdot h_b}{2}$$

$\vec{AC}(8,2,2)$

$$|\vec{AC}| = \sqrt{64+4+4} = \sqrt{72} = \sqrt{8 \cdot 9} = 6\sqrt{2}$$

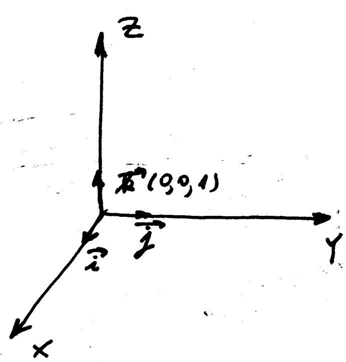
$$6\sqrt{2} \cdot h_b = 12\sqrt{3} \quad /: 6\sqrt{2}$$

$$h_b = 2\sqrt{\frac{3}{2}}$$

Površina trougla ΔABC je $6\sqrt{3}$ a visina koja odgovara stranici AC iznosi $2\sqrt{\frac{3}{2}}$.

3. Vektor $\vec{r}$ je normalan na Oz osu i na vektor $\vec{a}(8,-15,3)$. Ako je $|\vec{r}| = 51$ i $\measuredangle(\vec{r}, Ox)$ oštar, naći vektor $\vec{r}$.

Rj.



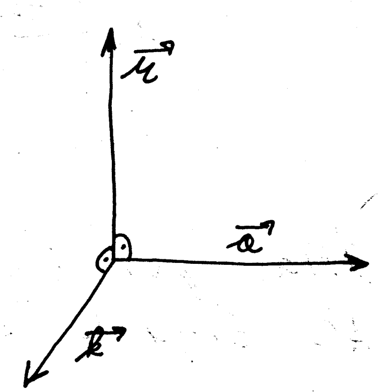
$\vec{r} \perp Oz$-osu

$\vec{r} \perp \vec{a}$

$|\vec{r}| = 51$

$\measuredangle(\vec{r}, Ox)$ oštar

$\vec{r} = ?$



$\Downarrow$

$\vec{r} \parallel \vec{a} \times \vec{k}$

tj.

$\vec{r} = \lambda(\vec{a} \times \vec{k})$

Stavimo $\vec{r}(x,y,z)$

$\vec{r} \perp Oz$-osa $\Rightarrow$ $\vec{r} \cdot \vec{k} = 0$

$$(x,y,z)(0,0,1) = 0+0+z$$

$$z = 0$$

$\vec{r} \perp \vec{a}$ $\Rightarrow$ $(x,y,0)(8,-15,3) = 0$

$$8x - 15y = 0$$

$$\vec{a}\times\vec{k}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 8 & -15 & 3\\ 0 & 0 & 1\end{vmatrix}=-15\vec{i}-8\vec{j}$$

$$\left.\begin{array}{l}\vec{r}=\lambda(-15,-8,0)=(-15\lambda,-8\lambda,0)\\ |\vec{r}|=51\end{array}\right\}\Longrightarrow \sqrt{225\lambda^2+64\lambda^2}=51$$

$$\sqrt{289\lambda^2}=51$$

$$17|\lambda|=51$$

$$|\lambda|=3$$

$$\lambda_1=-3 \quad \lambda_2=3$$

$\vec{r}_1\,(45,24,0)$

$\vec{r}_2\,(-45,-24,0)$

$\sphericalangle(\vec{r},O_x)$ oštar $\Rightarrow \cos\sphericalangle(\vec{r},O_x)>0$

tj. $\vec{r}\cdot\vec{i}>0$

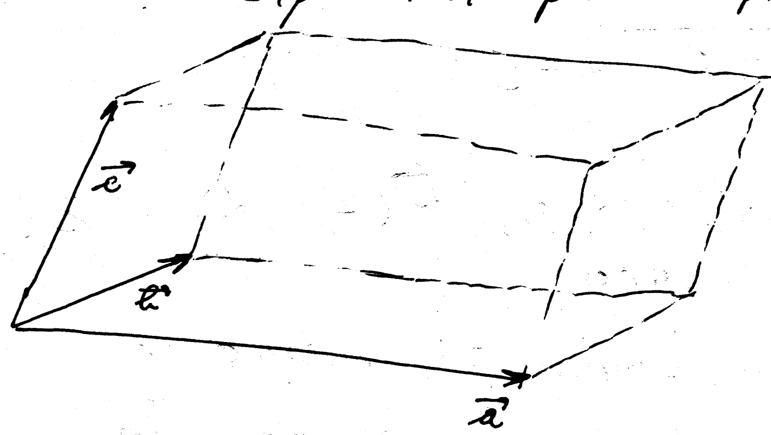
$\vec{r}\cdot\vec{i}=(x,y,z)(0,1,0)=y$

$y>0 \Rightarrow \vec{r}(45,24,0)$ traženi vektor

Mješoviti proizvod tri vektora

$$(\vec{a}\times\vec{b})\cdot\vec{c}=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

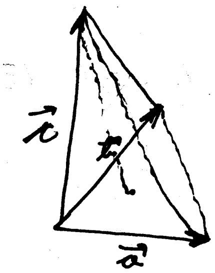
$(\vec{a}\times\vec{b})\cdot\vec{c}$ je broj koji je jednak zapremini paralelopipeda

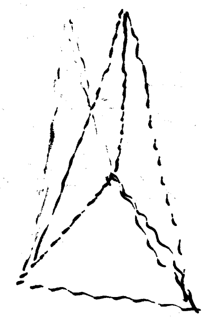


Ako je $(\vec{a}\times\vec{b})\cdot\vec{c}=0$. tada su $\vec{a}$, $\vec{b}$ i $\vec{c}$ komplanarni vektori

Zapremina tetraedra (piramide) kojeg obrazuju vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$

$$V = \frac{1}{6} \left| (\vec{a} \times \vec{b}) \cdot \vec{c} \right|$$





(1.) Provjeriti da li su vektori $\vec{a}(-1,3,2)$, $\vec{b}(2,-3,-4)$ i $\vec{c}(-3,12,6)$ komplanarni. Ako jesu izraziti vektor $\vec{c}$ preko vektora $\vec{a}$ i $\vec{b}$.

Rj: $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$ uslov komplanarnosti

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} -1 & 3 & 2 \\ 2 & -3 & -4 \\ -3 & 12 & 6 \end{vmatrix} \xlongequal[III_k + I_k \cdot 2]{II_k + I_k \cdot (3)} \begin{vmatrix} -1 & 0 & 0 \\ 2 & 3 & 0 \\ -3 & 3 & 0 \end{vmatrix} = 0$$

vektori su komplanarni

$$\vec{c} = \alpha \vec{a} + \beta \vec{b}$$

$$(-3, 12, 6) = \alpha(-1,3,2) + \beta(2,-3,-4)$$

$$\begin{aligned} -\alpha + 2\beta &= -3 \\ 3\alpha - 3\beta &= 12 \quad /:3 \\ 2\alpha - 4\beta &= 6 \quad /:(-2) \end{aligned}$$

$$\begin{aligned} -\alpha + 2\beta &= -3 \\ + \quad \alpha - \beta &= 4 \end{aligned}$$

$$\beta = 1 \qquad \alpha = 5$$

$$\vec{c} = 5\vec{a} + \vec{b}$$

vektor $\vec{c}$ razložen preko vektora $\vec{a}$ i $\vec{b}$

(2.) Vektori $\vec{a}(1, 2\alpha, 1)$, $\vec{b}(2, \alpha, \alpha)$ i $\vec{c}(3\alpha, 2, -\alpha)$ su ivice tetraedra

a) Odrediti zapreminu tog tetraedra

b) Odrediti α tako da $\vec{a}$, $\vec{b}$ i $\vec{c}$ budu komplanarni i u tom slučaju izraziti vektor $\vec{a}$ preko vektora $\vec{b}$ i $\vec{c}$.

Rj. $\vec{a}(1, 2\alpha, 1)$
$\vec{b}(2, \alpha, \alpha)$
$\vec{c}(3\alpha, 2, -\alpha)$

a) $V = \frac{1}{6} |(\vec{a} \times \vec{b}) \cdot \vec{c}|$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1 & 2\alpha & 1 \\ 2 & \alpha & \alpha \\ 3\alpha & 2 & -\alpha \end{vmatrix} \underset{II_k - III_k}{\overset{I_k - III_k}{=\!=}} \begin{vmatrix} 0 & 2\alpha-1 & 1 \\ 2-\alpha & 0 & \alpha \\ 4\alpha & 2+\alpha & -\alpha \end{vmatrix}$$

$$= -(2\alpha - 1) \begin{vmatrix} 2-\alpha & \alpha \\ 4\alpha & -\alpha \end{vmatrix} + \begin{vmatrix} 2-\alpha & 0 \\ 4\alpha & 2+\alpha \end{vmatrix} =$$

$$= (1-2\alpha) \begin{vmatrix} 2+3\alpha & 0 \\ 4\alpha & -\alpha \end{vmatrix} + (4-\alpha^2) = \overbrace{(1-2\alpha)(-\alpha) \cdot (2+3\alpha)}^{(2\alpha^2-\alpha)(3\alpha+2)} + 4 - \alpha^2 =$$

$$= 6\alpha^3 + \underline{4\alpha^2} \underline{-3\alpha^2} - 2\alpha + 4 \underline{-\alpha^2} = 2(3\alpha^3 - \alpha + 2)$$

$V = \frac{1}{3} |3\alpha^3 - \alpha + 2|$

zapremina tetraedra

b) $3\alpha^3 - \alpha + 2 = 0$

-1 je nula ovog polinoma pa

$$(3\alpha^3 - \alpha + 2) : (\alpha + 1) = 3\alpha^2 - 3\alpha + 2$$

$$\begin{array}{r} -\ \underline{3\alpha^3 + 3\alpha^2} \\ -3\alpha^2 - \alpha + 2 \\ -\ \underline{-3\alpha^2 - 3\alpha} \\ 2\alpha + 2 \\ \underline{2\alpha + 2} \\ // \quad // \end{array}$$

tj. $3\alpha^3 - \alpha + 2 = (\alpha+1)(3\alpha^2 - 3\alpha + 2)$

$(\alpha+1)\underbrace{(3\alpha^2 - 3\alpha + 2)} = 0$

$D = 9 - 24 < 0$
$a > 0$

$3\alpha^2 - 3\alpha + 2$ je uvijek pozitivno

$\Rightarrow \alpha = -1$

$\vec{a}(1, -2, 1)$
$\vec{b}(2, -1, -1)$
$\vec{c}(-3, 2, 1)$

$\vec{a} = \lambda \vec{b} + \mu \vec{c}$

$(1, -2, 1) = \lambda(2, -1, -1) + \mu(-3, 2, 1)$

$$\begin{aligned} 2\lambda - 3\mu &= 1 \\ -\lambda + 2\mu &= -2 \quad /\cdot 2 \\ -\lambda + \mu &= 1 \quad /\cdot 2 \end{aligned}$$

$$\begin{aligned} 2\lambda - 3\mu &= 1 \quad (1) \\ -2\lambda + 4\mu &= -4 \quad (2) \\ -2\lambda + 2\mu &= 2 \quad (3) \end{aligned}$$

(2)+(1):
(3)+(1): $\mu = -3 \Rightarrow \lambda = -4$

$\vec{a} = -4\vec{b} - 3\vec{c}$

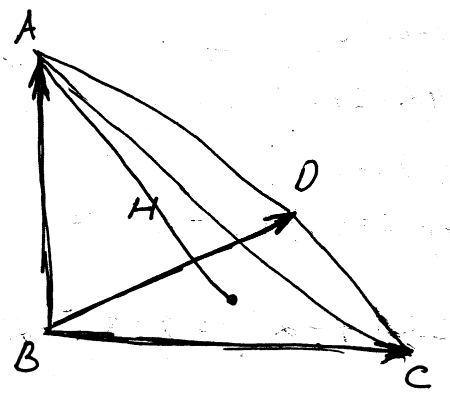
vektor $\vec{a}$ izražen preko $\vec{b}$ i $\vec{c}$

3. Date su tačke $A(3,2,1)$, $B(4,1,-2)$, $C(-5,-4,8)$ i $D(6,3,7)$. Odrediti:

a) zapreminu tetraedra $ABCD$.

b) visinu tetraedra koja odgovara osnovici BCD.

Rj.



$\left.\begin{array}{l} B(4,1,-2) \\ A(3,2,1) \end{array}\right\} \Rightarrow \overrightarrow{BA}(-1,1,3)$

$D(6,3,7) \Rightarrow \overrightarrow{BD}(2,2,9)$

$C(-5,-4,8) \Rightarrow \overrightarrow{BC}(-9,-5,10)$

a)
$$V=\frac{1}{6}\left|(\overrightarrow{BC}\times\overrightarrow{BD})\cdot\overrightarrow{BA}\right|=\frac{1}{6}\left|\begin{vmatrix} -9 & -5 & 10 \\ 2 & 2 & 9 \\ -1 & 1 & 3 \end{vmatrix}\right| \overset{I_k+II_k}{\underset{III_k-II_k\cdot 3}{=}} \frac{1}{6}\left|\begin{vmatrix} -14 & -5 & 25 \\ 4 & 2 & 3 \\ 0 & 1 & 0 \end{vmatrix}\right|$$

$$=\frac{1}{6}\left|\begin{vmatrix} -14 & 25 \\ 4 & 3 \end{vmatrix}\right| = \frac{1}{6}|-42-100| = \frac{142}{6} = \frac{71}{3}$$

Zapremina tetraedra $ABCD$ iznosi $\frac{71}{3}$.

b) Zapremina piramide $V=\frac{B\cdot H_{BCD}}{3}$

$$B=P_{\Delta ACD}=\frac{1}{2}\left|\overrightarrow{BC}\times\overrightarrow{BD}\right|=\frac{1}{2}\sqrt{4225+10\,201+64}=\frac{1}{2}\sqrt{9\cdot 1610}=\frac{3}{2}\sqrt{1610}$$

$$\overrightarrow{BC}\times\overrightarrow{BD}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -9 & -5 & 10 \\ 2 & 2 & 9 \end{vmatrix}=(-45-20)\vec{i}-(-81-20)\vec{j}+(-18+10)\vec{k}$$
$$=(-65,\ 101,\ -8)$$

$$\frac{71}{3}=\frac{\frac{3}{2}\sqrt{1610}\cdot H_{BCD}}{3} \qquad /\cdot 3\cdot 2$$

$$3\sqrt{1610}\cdot H_{BCD}=142$$

$$H_{BCD}=\frac{142}{3\sqrt{1610}}$$ je visina tetraedra koja odgovara osnovici BCD.

(#) Dati su vektori $\vec{a}\{\lambda, 3, 3\}$, $\vec{b}\{0, \lambda-1, \lambda+1\}$ i $\vec{c}\{\lambda, 3, 4\}$. Odrediti sve vrijednosti parametra λ tako da ovi vektori budu komplanarni, pa za veću vrijednost parametra λ razložiti vektor $\vec{a}$ preko vektora $\vec{b}$ i $\vec{c}$.

Rj.

Vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ su komplanarni akko $(\vec{a}\times\vec{b})\cdot\vec{c}=0$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} \lambda & 3 & 3 \\ 0 & \lambda-1 & \lambda+1 \\ \lambda & 3 & 4 \end{vmatrix} \underline{\underline{III_R - I_R}} \begin{vmatrix} \lambda & 3 & 3 \\ 0 & \lambda-1 & \lambda+1 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \lambda & 3 \\ 0 & \lambda-1 \end{vmatrix} =$$

$$= \lambda(\lambda-1)$$

$$\lambda(\lambda-1)=0$$

$$\lambda_1=0 \quad \lambda_2=1$$

Za vrijednost $\lambda=1$ vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ su komplanarni.

za $\lambda=1$ $\quad \vec{a}\{1, 3, 3\}$, $\vec{b}\{0, 0, 2\}$, $\vec{c}\{1, 3, 4\}$

$$\vec{a} = \alpha\vec{b} + \beta\vec{c}$$

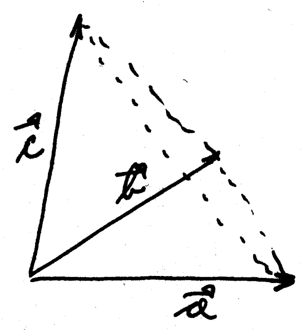
$$\{1, 3, 3\} = \alpha\{0, 0, 2\} + \beta\{1, 3, 4\}$$

$$\begin{aligned} 0\cdot\alpha + \beta &= 1 \\ 0\cdot\alpha + 3\beta &= 3 \\ 2\alpha + 4\beta &= 3 \\ \hline \beta &= 1 \end{aligned} \qquad \begin{aligned} 2\alpha + 4 &= 3 \\ 2\alpha &= -1 \\ \alpha &= -\frac{1}{2} \end{aligned}$$

$\vec{a} = -\frac{1}{2}\vec{b} + \vec{c}$ vektor $\vec{a}$ razložen preko vektora $\vec{b}$ i $\vec{c}$

Vektori $\vec{a}=(-1,-3,1)$, $\vec{b}=(\lambda,3,4)$ i $\vec{c}=(-5,-9,1)$ su ivice tetraedra. Odrediti parametar λ tako da zapremina tetraedra iznosi 8. Za vrijednost $\lambda=6$ provjeriti da li su vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ komplanarni; pa ako jesu izraziti vektor $\vec{a}$ preko vektora $\vec{b}$ i $\vec{c}$.

R:



$$V=\frac{1}{6}\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right| = \left|\frac{1}{6}\begin{vmatrix}-1 & -3 & 1\\ \lambda & 3 & 4\\ -5 & -9 & 1\end{vmatrix}\right| \ \frac{I_R-III_R}{II_R-III_R\cdot 4}$$

$$=\left|\frac{1}{6}\begin{vmatrix}4 & 6 & 0\\ \lambda+20 & 39 & 0\\ -5 & -9 & 1\end{vmatrix}\right| = \left|\frac{1}{6}\begin{vmatrix}4 & 6\\ \lambda+20 & 39\end{vmatrix}\right| =$$

$$=\left|\frac{1}{6}(156-6\lambda-120)\right| = \left|\frac{1}{6}(36-6\lambda)\right| = \left|\frac{1}{6}\cdot 6(6-\lambda)\right|$$

$V=|6-\lambda|$

$V=8 \quad \Rightarrow \quad \lambda=-2$ Za $\lambda=-2$ zapremina tetraedra iznosi 8.

Za vrijednost $\lambda=6$ zapremina tetraedra je 0 pa su vektori $\vec{a}=(-1,-3,1)$, $\vec{b}=(6,3,4)$ i $\vec{c}=(-5,-9,1)$ komplanarni.

$\vec{a}=\alpha\vec{b}+\beta\vec{c}$

$(-1,-3,1)=(6\alpha,3\alpha,4\alpha)+(-5\beta,-9\beta,\beta)$

$$\begin{aligned}6\alpha-5\beta&=-1\\ 3\alpha-9\beta&=-3 \quad /:3\\ 4\alpha+\beta&=1\end{aligned}$$

$$\begin{aligned}6\alpha-5\beta&=-1\\ \alpha-3\beta&=-1\\ 4\alpha+\beta&=1\end{aligned}$$

$$\begin{aligned}\alpha&=3\beta-1\\ 6\alpha-5\beta&=-1\\ 6(3\beta-1)-5\beta&=-1\\ 18\beta-6-5\beta&=-1\\ 13\beta&=5\\ \beta&=\frac{5}{13}\end{aligned}$$

$$\alpha=\frac{15}{13}-\frac{13}{13}$$

$$\alpha=\frac{2}{13}$$

$\vec{a}=\frac{2}{13}\vec{b}+\frac{5}{13}\vec{c}$ vektor $\vec{a}$ izražen preko vektora $\vec{b}$ i $\vec{c}$.

Zadaci za vježbu:

1. Kakav međusobni položaj zauzimaju vektori $\vec{a}$ i $\vec{b}$ ako je $|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$.

2. U trouglu $\triangle ABC$ data je tačka D na stranici BC tako da je $\overline{BD} = \frac{1}{3}\overline{BC}$, a na duži $\overline{AD}$ data je tačka E tako da je duž $\overline{AE} = \frac{1}{4}\overline{AD}$. Izračunati koordinate tačke C ako se zna da je $A(2,0,1)$, $B(-1,1,4)$ i $E(1,3,2)$.

3. Dati su vektori $\vec{u} = 6\vec{i}+\vec{j}+\vec{k}$, $\vec{v} = 3\vec{j}-\vec{k}$ i $\vec{w} = -2\vec{i}+3\vec{j}+5\vec{k}$. Odrediti x tako da $\vec{u}+x\vec{v} \perp \vec{w}$.

16. Koliki ugao obrazuju vektori $\vec{a}$ i $\vec{b}$ ako su vektori $5\vec{a}-3\vec{b} \perp 2\vec{a}+4\vec{b}$ i ako je $|\vec{a}| = 3$ i $|\vec{b}| = 2$.

17. Dokazati da se prave na kojima leže visine trougla sijeku u istoj tački.

18. Odrediti visinu h_B spuštenu iz vrha B u trouglu $\triangle ABC$ s vrhovima $A(1,-2,8)$, $B(0,0,4)$ i $C(6,2,0)$

19. Izračunati zapreminu paralelopipeda razapetog vektorima $\vec{a} = \vec{i}-3\vec{j}+\vec{k}$, $\vec{b} = 2\vec{i}+\vec{j}-3\vec{k}$ i $\vec{c} = \vec{i}+2\vec{j}+\vec{k}$.

20. Izračunati visinu paralelopipeda razapetog vektorima $\vec{a} = 3\vec{i}+2\vec{j}-5\vec{k}$, $\vec{b} = \vec{i}-\vec{j}+4\vec{k}$ i $\vec{c} = \vec{i}-3\vec{j}+\vec{k}$ ako je za osnovicu uzet paralelogram razapet vektorima $\vec{a}$ i $\vec{b}$.

21. Odredite α tako da zapremina tetraedra razapetog vektorima $\vec{a}$, $\vec{b}$ i $\alpha\vec{c}$ iznosi $\frac{2}{3}$, gdje je $\vec{a} = \vec{i}+\vec{j}-2\vec{k}$, $\vec{b} = 2\vec{i}+\vec{j}-\vec{k}$ i $\vec{c} = \vec{i}-\frac{1}{3}\vec{k}$.

22. Zadan je trokut s vrhovima $A(2,3,2)$, $B(0,1,1)$ i $C(4,4,0)$. Odredite koordinate tačke S presjeka simetrale unutrašnjeg ugla pri vrhu A i simetrale stranice AB.

23. Dokažite vektorskim računom da se u trouglu simetrale stranica sijeku u jednoj tački.